Study of η_c and η_c' decays into vector meson pairs

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The processes of $\eta_c(\eta_c') \to VV$ are supposed to be suppressed by the helicity selection rule (HSR) but found to be rather important decay modes in experiment for η_c. We try to distinguish the short-distance transitions via the singly disconnected Okubo-Zweig-Iizuka (SOZI) processes and the doubly disconnected (DOZI) processes in $\eta_c(\eta_c') \to VV$. It shows that the SOZI processes can be related to the $\eta_c(\eta_c')$ wavefunctions at the origin. Therefore, a relation of the decay branching ratio fraction between these two decay channels can be established. Such a relation is similar to that for J/ψ and $\psi' \to VP$, where the so-called "$\rho\pi$ puzzle" has been existed for a while. We also show that the intermediate charmed meson loop transitions provide an evading mechanism for the DOZI processes. This contribution would turn out to be more important in $\eta_c' \to VV$. As a consequence, it may produce significant deviations from the SOZI-dominant scenario. Future experimental measurement of the $\eta_c' \to VV$ by BESIII should be able to clarify the DOZI-evading mechanisms.

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I. INTRODUCTION

Among the charmonium states below $D\bar{D}$ threshold, there is little information on the η_c' decays from experiment. Even its mass and width still have large uncertainties as listed in the Particle Data Group [1]. From a theoretical view point, this state contains rich information on the dynamics in the interplay of perturbative and non-perturbative QCD, for which there are still a lot of questions unanswered.

Our motivation of studying η_c and η_c' simultaneously is driven by two puzzling but interesting questions. Firstly, it has been observed that $\eta_c \to VV$, where V stands for light vector meson, is one of the most important decay channels for η_c with branching ratios at the order of 10^{-3} to 10^{-2}. However, these decay channels as well as $\eta_c' \to VV$ are supposed to be highly suppressed by the so-called helicity selection rule (HSR) [2–4]. Such an observation of the HSR violation indicates the importance of QCD higher twist contributions or presence of a non-pQCD mechanism that violates the HSR. The contradiction between the data and the HSR expectations based on the perturbative QCD (pQCD) has drawn some attention, and various attempts have been made to try to understand the underlying dynamics [5–13]. In Refs. [14, 15], we studied the effects of charmed hadron loops as a source of long distance contribution which violates the HSR. The results indeed suggest that contributions from intermediate charmed hadron loops are significant and can be taken as an evading mechanism of the HSR. In $\eta_c(\eta_c') \to VV$ it is natural to expect that the charmed hadron loops would contribute and might provide an evading mechanism here. Even for higher energy processes such as $\eta_b \to J/\psi J/\psi$, it was also found that charmed meson loops would enhance the decay rate significantly [11]. In Ref. [16], possible contributions from charmed meson loops to $e^+e^- \to J/\psi\eta_c$ at $W = 10.56$ GeV were also investigated.

The second reason that we are interested in $\eta_c(\eta_c')$ exclusive decays is that they are related with the long-standing so-called "$\rho\pi$ puzzle" in $J/\psi(\psi') \to VP$. The decays of J/ψ and ψ' into light hadrons are supposed to be via the valence $c\bar{c}$ annihilations into three gluons in pQCD to the leading order at a typical distance of $1/m_c$. In the heavy quark limit, i.e. m_c is infinitely large, the decay amplitude will be proportional to the wave function at the origin $\psi(0)$ ($\psi'(0)$). As a result the following relation is expected for the inclusive final states of light hadrons h,

$$R_{\psi\psi'} \equiv \frac{BR(\psi' \to h)}{BR(J/\psi \to h)} = \frac{BR(\psi' \to e^+e^-)}{BR(J/\psi \to e^+e^-)} \simeq \left|\frac{\psi'(0)}{\psi(0)}\right|^2 \frac{\Gamma^{\psi}_{tot}}{\Gamma^{\psi'}_{tot}} \simeq 12\%, \tag{1}$$

where the phase space factors are taken into account by Γ_{tot} in the heavy quark limit. For the inclusive decay and many exclusive decays, the data exhibit consistencies with this relation rather well, which is called "12% rule". However, some exclusive decay modes, such as $\rho\pi$ and $K^*\bar{K} + c.c.$, are found largely violating the "12% rule", which is known as the "$\rho\pi$ puzzle" problem.

Many theoretical efforts have been made in order to understand the origin of such a significant deviation from the "12% rule" in $J/\psi(\psi') \to \rho\pi$ and $K^*\bar{K} + c.c.$ In recent works [17, 18], we show that the interferences between the strong and EM decay amplitudes in both $J/\psi(\psi') \to VP$ are essential for understanding the "$\rho\pi$ puzzle". Similar ideas had been proposed in the literature [19, 20]. However, if this is the case, one has to clarify and even quantify

the mechanism which suppressed the strong transition amplitude in $\psi' \to VP$ and then makes it to be compatible with the EM amplitudes in some channels. A numerical study of the overall decay channels for $J/\psi(\psi') \to VP$ indeed suggests such a phenomenon [18]. In Refs. [18, 21], it was shown that the intermediate charmed meson loops, which serves as a long-distance mechanism for both OZI rule and HSR evasions, can significantly suppress the $\psi' \to VP$ strong transition amplitudes. Such a mechanism will largely alter the branching ratios for $J/\psi(\psi') \to VP$, and makes the relation in Eq. (1) unreliable.

In fact, the intermediate charmed meson loops could be much more general than we would expect in charmonium energy region. Further studies of the effects from intermediate meson loops in other processes have been reported in Refs. [14, 15, 22–25].

The coherent study of the η_c and $\eta_c' \to VV$ will be useful for clarifying the role played by the intermediate meson loops and EM transition amplitudes in $J/\psi(\psi') \to VP$. Since η_c and η_c' are just the spin 0 partners of J/ψ and ψ' respectively, and they may possess the same spatial wave functions in the heavy quark limit, we would expect a similar relation as Eq.(1) to hold between η_c and η_c', namely,

$$R_{\eta_c\eta_c'} \equiv \frac{BR(\eta_c' \to h)}{BR(\eta_c \to h)} = \frac{BR(\eta_c' \to \gamma\gamma)}{BR(\eta_c \to \gamma\gamma)} \simeq \left|\frac{\eta_c'(0)}{\eta_c(0)}\right|^2 \frac{\Gamma^{\eta_c}_{tot}}{\Gamma^{\eta_c'}_{tot}} \simeq 0.52 \sim 1.56, \tag{2}$$

where we have taken $\eta_c'(0)/\eta_c(0) = \psi'(0)/\psi(0) = 0.64$ [26], and the range of the ratio is displayed considering the larger uncertainties of the η_c' total width [1]. Alternatively, we can express this relation as

$$\bar{R}_{\eta_c\eta_c'} \equiv \frac{\Gamma_{\eta_c'\to h}}{\Gamma_{\eta_c\to h}} \simeq \left|\frac{\eta_c'(0)}{\eta_c(0)}\right|^2 \simeq 0.41 \ , \tag{3}$$

which avoids the uncertainties with the total width of η_c'.

In exclusive decays of $\eta_c(\eta_c') \to VV$, as discussed earlier the transition amplitudes should be suppressed by the HSR. However, since the mass of the charm quark is not heavy enough and due to the non-vanishing light quark masses, the HSR violation should occur via both singly disconnected Okubo-Zweig-Iizuka (SOZI) transitions and doubly disconnected (DOZI) processes. The latter can be related to long-distance hadronic transitions via intermediate charmed meson loops [22]. For the SOZI transitions, the relation of Eq. (3) will be respected due to their short-distance feature, while the long-distance charmed meson loops will not necessarily respect it. In parallel with the observations in $J/\psi(\psi') \to VP$, if the charmed meson loops play a more significant role in $\eta_c' \to VV$ than in $\eta_c \to VV$, they may violate the relation of Eq. (3) and lead to observable phenomena similar to the "$\rho\pi$ puzzle" in these two decay channels.

Our focus in this work is to investigate the role played by the intermediate charmed meson loops in $\eta_c(\eta_c') \to VV$. Also, we mention in advance that due to lack of experimental information to constrain the relative strength between SOZI and loop transition amplitudes, some of the results have to be qualitative. But they can be examined by the forthcoming BESIII high-statistics experiment.

As follows, in Sec. II, we discuss the parametrization scheme for $\eta_c(\eta_c') \to VV$ and constraints from the available experimental data. The intermediate charmed meson loops are described by an effective Lagrangian approach in Sec. III, and numerical results are presented. A brief summary is given in Sec. IV.

II. PARAMETRIZATION FOR $\eta_c(\eta_c') \to VV$

An obvious advantage for the exclusive decays of $\eta_c(\eta_c') \to VV$ is that these transitions, similar to $J/\psi(\psi') \to VP$, have only one unique Lorentz structure for the VVP couplings. As stressed a number of times before, this will allow a parametrization of the effective coupling constant contributed by different mechanisms. This will help a lot especially at this moment when the experimental data for η_c' are not available.

In Ref. [8], a parametrization scheme is proposed to distinguish the SOZI and DOZI processes by the gluon counting rule. Due to the above mentioned property with the VVP coupling, one can factorize out the DOZI evading amplitude, of which the ratio to the SOZI amplitude will give an estimate of the order of magnitude of the contributions from the DOZI mechanisms.

Following Ref. [8], the transition amplitudes for $\eta_c \to VV$ can be expressed as

$$\begin{aligned}
\langle\phi\phi|\hat{V}_{gg}|\eta_c\rangle &= g_0^2 R^2(1+r) \\
\langle\omega\omega|\hat{V}_{gg}|\eta_c\rangle &= g_0^2(1+2r) \\
\langle\omega\phi|\hat{V}_{gg}|\eta_c\rangle &= g_0^2 rR\sqrt{2}
\end{aligned}$$

$$\begin{aligned}\langle K^{*+}K^{*-}|\hat{V}_{gg}|\eta_c\rangle &= g_0^2 R \\ \langle \rho^+\rho^-|\hat{V}_{gg}|\eta_c\rangle &= g_0^2 \ .\end{aligned} \tag{4}$$

where $\hat{V}_{gg}$ is the $\eta_c \to gg \to (q\bar{q})(q\bar{q})$ potential, and parameter g_0 denotes the coupling strength of the SOZI transitions. Parameter r is the ratio of the DOZI transition over the SOZI transition. It should be pointed out that the additional gluon exchange in DOZI is not necessarily perturbative. Due to contributions from effective mesonic degrees of freedom, namely, intermediate meson exchanges, the DOZI transitions can be evaded by rather soft gluon exchanges of which the contributions can be parameterized by r. We also introduce the SU(3) flavor breaking parameter R, of which its deviation from unity reflects the change of couplings due to the mass difference between u/d and s. The amplitudes for other charge combinations of $K^*\bar{K}^*$ and $\rho\rho$ are implicated.

A commonly used form factor is adopted in the calculation of the partial decay widths:

$$\mathcal{F}^2(\mathbf{p}) = p^{2l}\exp(-\mathbf{p}^2/8\beta^2) \ , \tag{5}$$

where $\mathbf{p}$ and l are the three momentum and relative angular momentum of the final-state mesons, respectively, in the η_c rest frame. We adopt $\beta = 0.5$ GeV, which is the same as in Refs. [27–30]. Such a form factor will largely account for the size effects from the spatial wavefunctions of the initial and final state mesons.

The present experimental situation should be clarified. In 2005, BESII Collaboration measured the exclusive decay branching ratios of $\eta_c \to VV$ [31], where they find significant differences from the DM2 results [32] in $\eta_c \to \rho\rho$. In fact, the PDG averaged value for $\eta_c \to \rho\rho$ branching ratio is strongly affected by this discrepancy. As a comparison of the BESII results and the PDG averaged values, we list in Table I the fitting results of BESII data (Fit-II in Ref. [8]) and PDG2008 [1]. The fitted parameters are listed in Table II.

BR ($\times 10^{-3}$)	BESII [31]	Fit-BES	PDG2008 [1]	Fit-PDG
$\rho\rho$	$12.5 \pm 3.7 \pm 5.1$	10.7	20.0 ± 7.0	11.80
$K^*\bar{K}^*$	$10.4 \pm 2.6 \pm 4.3$	13.6	9.2 ± 3.4	11.76
$\phi\phi$	$2.5 \pm 0.5 \pm 0.9$	2.16	2.7 ± 0.9	2.23
$\omega\omega$	< 6.3	1.67	< 3.1	4.53
$\omega\phi$	< 1.3	0.33	< 1.7	0.02

TABLE I: The branching ratios for $\eta_c \to VV$. The data are from BES. Fit-BES is obtained by fitting the BESII data [31] for $\eta_c \to \phi\phi$, $K^*\bar{K}^*$ and $\rho\rho$, while Fit-PDG are obtained by fitting the PDG2008 data [1].

Parameters	Fit-BES	Fit-PDG
r	-0.16 ± 0.15	0.04 ± 0.16
R	1.02 ± 0.23	0.91 ± 0.16
g_0	0.35 ± 0.04	0.36 ± 0.04
χ^2	0.5	2.4

TABLE II: The parameters determined in Fit-BES and Fit-PDG.

It is interesting to read that the two fitting results are rather consistent with each other in $\eta_c \to \rho\rho$, $K^*\bar{K}^*$ and $\phi\phi$. Also, the fitted parameters agree well in these two fits except that the χ^2 has a much larger value in Fit-PDG. The fitted branching ratio for $\rho\rho$ has significant discrepancies with the PDG averaged value. Differences between these two sets of data lead to a relatively large uncertainty with parameter r, while parameters g_0 and R are rather stable. It suggests that the DOZI-evading mechanism will only account for an order of $r^2 \simeq 0 \sim 0.1$ in $\eta_c \to VV$, and the SOZI process should be dominant. This observation is within our expectation and consistent with what we find in $J/\psi(\psi') \to VP$, where the DOZI-evading mechanism in $J/\psi \to VP$ is negligibly small [18].

The fitting results also show that the present experimental uncertainties are rather large, and the DOZI-evading mechanism cannot be well constrained by the data. It should be pointed out that the relation of Eq. (3) may help estimate the SOZI contributions in $\eta_c' \to VV$ given the dominance of the SOZI processes. However, due to lack of data for η_c', it is not possible to estimate the DOZI evading contributions in $\eta_c' \to VV$ based on the parametrization scheme.

In order to proceed, we assume that the DOZI-evading mechanisms are via the intermediate charmed meson loops, for which a quantitative study of the effects can be quantified. By taking the upper limit of the DOZI contributions in $\eta_c \to VV$, we can then estimate the possible impact of the DOZI-evading mechanisms in $\eta_c(\eta_c') \to VV$. In particular, we would like to examine whether the relation of Eq. (3) still holds or not with the presence of the DOZI contributions.

III. DOZI-EVADING MECHANISMS VIA INTERMEDIATE CHARMED MESON LOOPS

A. Formulation

We will use an effective Lagrangian approach to estimate the transition amplitudes. In Figs. 1, 2, 3 and 4, the Feynman diagrams of η_c decaying to $\rho\rho$, $K^*\bar{K}^*$, $\omega\omega$, and $\phi\phi$ via the intermediate charmed meson loops are presented respectively. The relevant Lagrangians based on heavy quark symmetry which describes the coupling between S-wave charmonium and the charmed mesons reads [33, 34],

$$\mathcal{L}_2 = ig_2 Tr[R_{c\bar{c}} \bar{H}_{2i} \gamma^\mu \overleftrightarrow{\partial}_\mu \bar{H}_{1i}] + H.c., \tag{6}$$

where the S-wave charmonium states are expressed as

$$R_{c\bar{c}} = \left(\frac{1+\not{v}}{2}\right)(\psi^\mu \gamma_\mu - \eta_c \gamma_5)\left(\frac{1-\not{v}}{2}\right). \tag{7}$$

And the charmed and anti-charmed meson triplet are

$$\begin{aligned} H_{1i} &= \left(\frac{1+\not{v}}{2}\right)[\mathcal{D}_i^{*\mu}\gamma_\mu - \mathcal{D}_i\gamma_5], &(8)\\ H_{2i} &= [\bar{\mathcal{D}}_i^{*\mu}\gamma_\mu - \bar{\mathcal{D}}_i\gamma_5]\left(\frac{1-\not{v}}{2}\right), &(9)\end{aligned}$$

where $\mathcal{D}$ and $\mathcal{D}^*$ are pseudoscalar ((D^0, D^+, D_s^+)) and vector charmed mesons ($(D^{*0}, D^{*+}, D_s^{*+})$), respectively. The Lagrangian describing the interactions between light mesons and charmed mesons reads

$$\begin{aligned} \mathcal{L} &= -ig_{\rho\pi\pi}\left(\rho_\mu^+ \pi^0 \overleftrightarrow{\partial}^\mu \pi^- + \rho_\mu^- \pi^+ \overleftrightarrow{\partial}^\mu \pi^0 + \rho_\mu^0 \pi^- \overleftrightarrow{\partial}^\mu \pi^+\right) \\ &- ig_{\mathcal{D}^*\mathcal{D}\mathcal{P}}(\mathcal{D}^i \partial^\mu \mathcal{P}_{ij} \mathcal{D}_\mu^{*j\dagger} - \mathcal{D}_\mu^{*i} \partial^\mu \mathcal{P}_{ij} \mathcal{D}^{j\dagger}) + \frac{1}{2} g_{\mathcal{D}^*\mathcal{D}^*\mathcal{P}} \epsilon_{\mu\nu\alpha\beta} \mathcal{D}_i^{*\mu} \partial^\nu \mathcal{P}^{ij} \overleftrightarrow{\partial}^\alpha \mathcal{D}_j^{*\beta\dagger} \\ &- ig_{\mathcal{D}\mathcal{D}\mathcal{V}} \mathcal{D}_i^\dagger \overleftrightarrow{\partial}_\mu \mathcal{D}^j (V^\mu)_j^i - 2f_{\mathcal{D}^*\mathcal{D}\mathcal{V}} \epsilon_{\mu\nu\alpha\beta} (\partial^\mu \mathcal{V}^\nu)_j^i (\mathcal{D}_i^\dagger \overleftrightarrow{\partial}^\alpha \mathcal{D}^{*\beta j} - \mathcal{D}_i^{*\beta\dagger} \overleftrightarrow{\partial}^\alpha D^j) \\ &+ ig_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} \mathcal{D}_i^{*\nu\dagger} \overleftrightarrow{\partial}_\mu \mathcal{D}_\nu^{*j} (\mathcal{V}^\mu)_j^i + 4if_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} \mathcal{D}_{i\mu}^{*\dagger} (\partial^\mu \mathcal{V}^\nu - \partial^\nu \mathcal{V}^\mu)_j^i \mathcal{D}_\nu^{*j}, \end{aligned} \tag{10}$$

with the convention $\epsilon^{0123} = +1$, where $\mathcal{P}$ and $\mathcal{V}_\mu$ denote 3×3 matrices for the pseudoscalar octet and vector nonet mesons respectively [35],

$$\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \quad \mathcal{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \tag{11}$$

The following kinematic conventions are adopted, $\eta_c(p) \to \mathcal{D}^{(*)}(p_1)\bar{\mathcal{D}}^{(*)}(p_3)[\mathcal{D}^{(*)}(p_2)] \to V(k)V(q)$, where $\mathcal{D}^{(*)}$ in the square bracket denotes the exchanged charmed meson in the triangle diagrams. There are four types of transition amplitudes corresponding to different kinds of charmed meson exchanges in the triangle diagrams. We take the amplitudes of $\eta_c \to \phi\phi$ as an example,

$$\begin{aligned} \mathcal{M}_{\mathcal{D}\mathcal{D}^*[\mathcal{D}]} &= \int \frac{d^4 p_1}{(2\pi)^4}[-2g_{\eta_c\mathcal{D}\mathcal{D}^*} g_{\mathcal{D}\mathcal{D}\mathcal{V}} f_{\mathcal{D}\mathcal{D}^*\mathcal{V}} \epsilon_{\mu\nu\alpha\beta}(p_1+p_2)\cdot\epsilon_k q^\mu \epsilon_q^\nu \\ &\times (p_2-p_3)^\alpha (p_1-p_3)_\lambda (-g^{\lambda\beta} + \frac{p_3^\lambda p_3^\beta}{m_3^2})]\frac{1}{a_1 a_2 a_3}\mathcal{F}(p_i^2), \end{aligned} \tag{12}$$

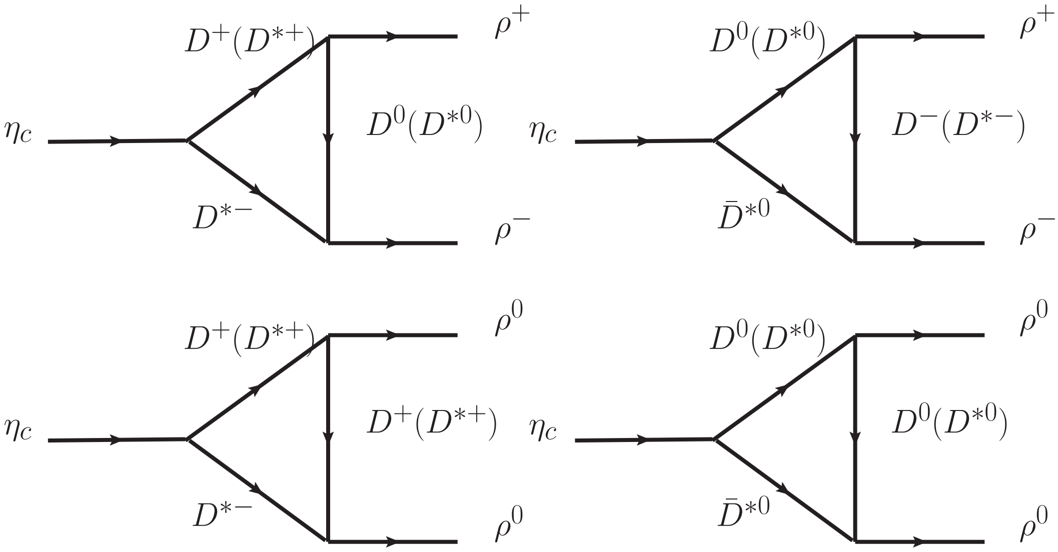

FIG. 1: Feynman diagrams for $\eta_c \to \rho\rho$ via intermediate charmed meson loops.

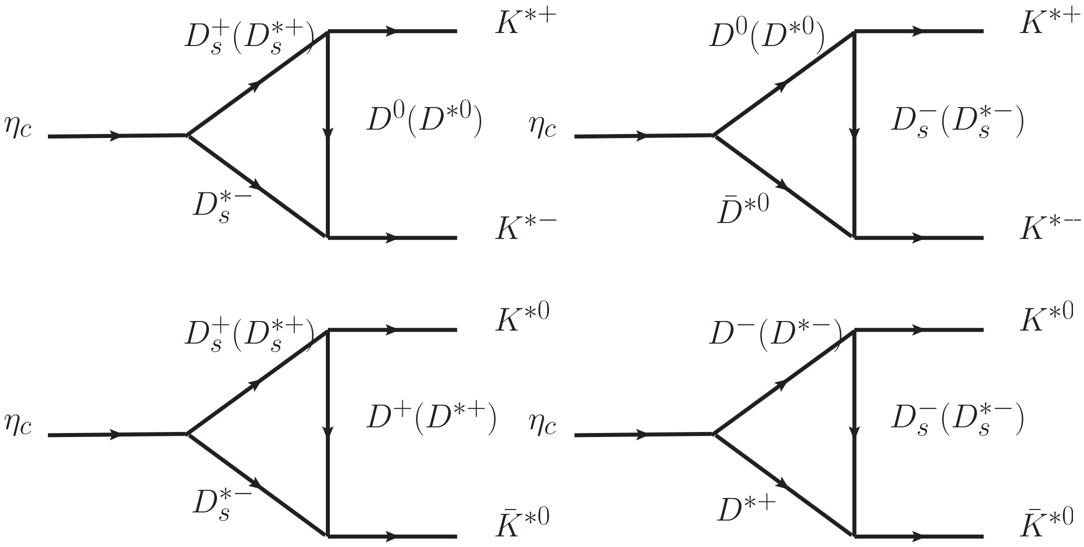

FIG. 2: Feynman diagrams for $\eta_c \to K^*\bar{K}^*$ via intermediate charmed meson loops.

$$
\begin{aligned}
\mathcal{M}_{\mathcal{D}\mathcal{D}^*[\mathcal{D}^*]} &= \int \frac{d^4p_1}{(2\pi)^4}[-8g_{\eta_c\mathcal{D}\mathcal{D}^*}f_{\mathcal{D}\mathcal{D}^*\mathcal{V}}f_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}k^\mu\epsilon_k^\nu(p_1+p_2)^\alpha \\
&\times (-g^{\beta\lambda}+\frac{p_2^\beta p_2^\lambda}{m_2^2})(p_1-p_3)_\delta(-g^{\delta\theta}+\frac{p_3^\delta p_3^\theta}{m_3^2})(\epsilon_{q\lambda}q_\theta-q_\lambda\epsilon_{q\theta}) \\
&+ 2g_{\eta_c\mathcal{D}\mathcal{D}^*}f_{\mathcal{D}\mathcal{D}^*\mathcal{V}}g_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}k^\mu\epsilon_k^\nu(p_1+p_2)^\alpha(p_2-p_3)\cdot\epsilon_q \\
&\times (p_1-p_3)_\delta(-g^{\beta\lambda}+\frac{p_2^\beta p_2^\lambda}{m_2^2})(-g_\lambda^\delta+\frac{p_{3\lambda}p_3^\delta}{m_3^2})]\frac{1}{a_1a_2a_3}\mathcal{F}(p_i^2),
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
\mathcal{M}_{\mathcal{D}^*\mathcal{D}^*[\mathcal{D}]} &= \int \frac{d^4p_1}{(2\pi)^4}[-4g_{\eta_c\mathcal{D}^*\mathcal{D}^*}f^2_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}\epsilon_{\rho\sigma\lambda\iota}\epsilon_{\eta\theta\xi\omega}p^\nu p_1^\mu(p_1+p_2)^\lambda \\
&\times k^\rho\epsilon_k^\sigma q^\eta\epsilon_q^\theta(p_2-p_3)^\xi(-g^{\beta\iota}+\frac{p_1^\beta p_1^\iota}{m_1^2})(-g^{\alpha\omega}+\frac{p_3^\alpha p_3^\omega}{m_3^2})]\frac{1}{a_1a_2a_3}\mathcal{F}(p_i^2),
\end{aligned}
\tag{14}
$$

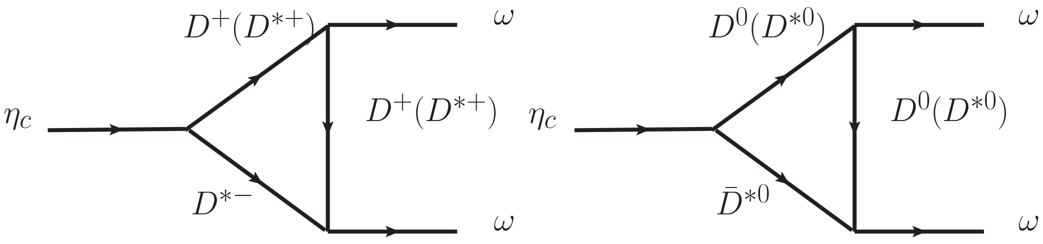

FIG. 3: Feynman diagrams for $\eta_c \to \omega\omega$ via intermediate charmed meson loops.

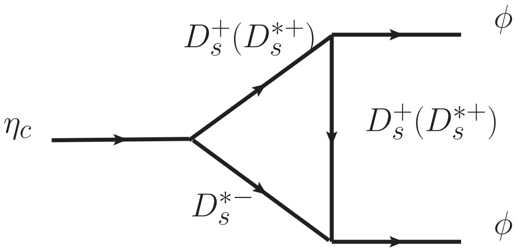

FIG. 4: Feynman diagrams for $\eta_c \to \phi\phi$ via intermediate charmed meson loops.

$$
\begin{aligned}
\mathcal{M}_{\mathcal{D}^*\mathcal{D}^*[\mathcal{D}^*]} &= \int \frac{d^4 p_1}{(2\pi)^4}[\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4]\frac{1}{a_1 a_2 a_3}\mathcal{F}(p_i^2), \qquad (15)\\
\mathcal{A}_1 &= g_{\eta_c \mathcal{D}^*\mathcal{D}^*} g^2_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} \epsilon_{\mu\nu\alpha\beta} p^\nu p_1^\mu (p_1 + p_2)\cdot\epsilon_k (p_2 - p_3)\cdot\epsilon_q \\
&\times\ (-g^{\beta\lambda} + \frac{p_1^\beta p_1^\lambda}{m_1^2})(-g_{\lambda\delta} + \frac{p_{2\lambda}p_{2\delta}}{m_2^2})(-g^{\delta\alpha} + \frac{p_3^\delta p_3^\alpha}{m_3^2})\\
\mathcal{A}_2 &= -4 g_{\eta_c \mathcal{D}^*\mathcal{D}^*} g_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} f_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} \epsilon_{\mu\nu\alpha\beta} p^\nu p_1^\mu (p_1 + p_2)\cdot\epsilon_k \\
&\times\ (-g^{\beta\lambda} + \frac{p_1^\beta p_1^\lambda}{m_1^2})(-g_{\lambda\rho} + \frac{p_{2\lambda}p_{2\rho}}{m_2^2})(-g^\alpha_\delta + \frac{p_{3\delta} p_3^\alpha}{m_3^2})(\epsilon_q^\rho q^\delta - q^\rho \epsilon_q^\delta)\\
\mathcal{A}_3 &= -4 g_{\eta_c \mathcal{D}^*\mathcal{D}^*} g_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} f_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} \epsilon_{\mu\nu\alpha\beta} p^\nu p_1^\mu (-p_3 + p_2)\cdot\epsilon_q \\
&\times\ (-g^\beta_\lambda + \frac{p_1^\beta p_{1\lambda}}{m_1^2})(-g_{\sigma\rho} + \frac{p_{2\sigma}p_{2\rho}}{m_2^2})(-g^{\rho\alpha} + \frac{p_3^\rho p_3^\alpha}{m_3^2})(k^\sigma \epsilon_k^\lambda - \epsilon_k^\sigma k^\lambda)\\
\mathcal{A}_4 &= 16 g_{\eta_c \mathcal{D}^*\mathcal{D}^*} f^2_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} \epsilon_{\mu\nu\alpha\beta} p^\nu p_1^\mu (-g^\beta_\lambda + \frac{p_1^\beta p_{1\lambda}}{m_1^2})(-g^\alpha_\rho + \frac{p_2^\alpha p_{2\rho}}{m_2^2})\\
&\times\ (-g_{\theta\delta} + \frac{p_{3\theta}p_{3\delta}}{m_3^2})(q^\rho \epsilon_k^\lambda k^\theta \epsilon_q^\delta + \epsilon_q^\rho k^\lambda q^\theta \epsilon_k^\delta - \epsilon_q^\rho \epsilon_k^\lambda q^\theta k^\delta - q^\rho k^\lambda \epsilon_k^\theta \epsilon_q^\delta)
\end{aligned}
$$

where $a_1 \equiv p_1^2 - m_1^2$, $a_2 \equiv p_2^2 - m_2^2$, and $a_3 \equiv p_3^2 - m_3^2$. The amplitudes of other processes have similar forms as the above. We omit them for the sake of brevity.

Since the couplings in the effective Lagrangians are local ones, ultra-violate divergence in the loop integrals is inevitable. We introduce a form factor $\mathcal{F}(p_i^2)$ phenomenologically to take into account the non-local effects and cut off the divergence in the loop integrals, i.e.

$$
\mathcal{F}(p_i^2) = \prod_i \left(\frac{\Lambda_i^2 - m_i^2}{\Lambda_i^2 - p_i^2}\right), \qquad (16)
$$

where m_i and p_i are the mass and four momentum of the corresponding exchanged particle, and the cut-off energy is chosen as $\Lambda_i = m_i + \alpha\Lambda_{QCD}$ with $\Lambda_{QCD} = 0.22$ GeV [14, 15, 35]. The value of parameter α is commonly expected to be of order of unity.

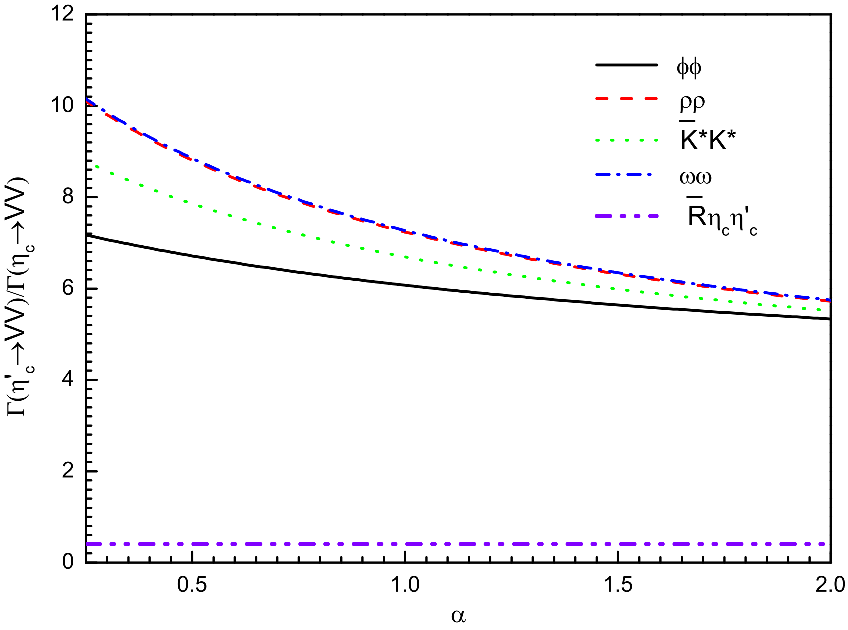

FIG. 5: The partial width fractions $\Gamma^{loop}_{\eta_c'\to VV}/\Gamma^{loop}_{\eta_c\to VV}$ in terms of the parameter α. The ratio of $\bar{R}_{\eta_c\eta_c'}$, which is independent of α, is also displayed by the dot-dot-dashed line.

B. Numerical results for DOZI-evading contributions

Before proceeding to the numerical results, we first determine some of the parameters taken in this approach. In the chiral and heavy quark limit, the following relations can be obtained [34, 35],

$$g_{\mathcal{DDV}} = g_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} = \frac{\beta g_{\mathcal{V}}}{\sqrt{2}},\ f_{\mathcal{DD}^*\mathcal{V}} = \frac{f_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}}{m_{\mathcal{D}^*}} = \frac{\lambda g_{\mathcal{V}}}{\sqrt{2}},\ g_{\mathcal{V}} = \frac{m_\rho}{f_\pi}, \tag{17}$$

$$g_{\eta_c\mathcal{DD}^*} = g_{\eta_c\mathcal{D}^*\mathcal{D}^*}\sqrt{\frac{m_{\mathcal{D}}}{m_{\mathcal{D}^*}}}m_{\eta_c} = 2g_2\sqrt{m_{\eta_c}m_{\mathcal{D}}m_{\mathcal{D}^*}}, \tag{18}$$

where β and λ are commonly taken as $\beta = 0.9, \lambda = 0.56$ GeV^{-1}, while f_π is the pion decay constant.

In principle, the coupling g_2 should be computed by nonperturbative methods. If we simply estimate the coupling g_2 with the vector meson dominance (VMD) argument, it will give $g_2 = \sqrt{m_\psi}/(2m_{\mathcal{D}}f_\psi)$, where m_ψ and $f_\psi = 405$ MeV being the mass and decay constant of J/ψ [33]. This relation gives $g_{\eta_c\mathcal{DD}^*} = 7.68$, which is a commonly adopted value in the literature. In order to determine $g_{\eta_c'\mathcal{D}^{(*)}\mathcal{D}^{(*)}}$, we similarly relate it to the coupling of $g_{\psi'\mathcal{DD}} = 9.05$ [36], which is determined by the experimental data for $e^+e^- \to D\bar{D}$ [37]. In Ref. [38], similar values for $g_{\eta_c\mathcal{D}^{(*)}\mathcal{D}^{(*)}}$ and $g_{\eta_c'\mathcal{D}^{(*)}\mathcal{D}^{(*)}}$ were adopted.

The determination of the form factor parameter α will depend on the relative strengths and interferences between the SOZI and DOZI-evading amplitudes. As discussed in the previous Section, due to lack of sufficient experimental information about the $\eta_c' \to VV$, we can only estimate the upper limit of the DOZI-evading contributions based on the data for $\eta_c \to VV$. Interestingly, we find that for the same value of the form factor parameter α, the ratio $\Gamma^{loop}_{\eta_c'\to VV}/\Gamma^{loop}_{\eta_c\to VV}$ is rather insensitive to α of a broad range. As mentioned before, the advantage of taking this ratio is that the uncertainties of the η_c' total width can be avoided. In Fig. 5, the ratio $\Gamma^{loop}_{\eta_c'\to VV}/\Gamma^{loop}_{\eta_c\to VV}$ in terms of a common range of α is displayed. The dot-dot-dashed line (independent of α) denotes the ratio given by Eq. (3), which turns to be much smaller than the ratio between the charmed meson loops.

The following points can be learned:

$\Gamma^{loop}(\times 10^{-2})$ MeV	$\eta_c \to VV$	$\eta_c' \to VV$
$\phi\phi$	$0.48 \sim 0.96$	$3.10 \sim 6.05$
$\rho\rho$	$3.50 \sim 6.87$	$28.04 \sim 53.70$
$K^*\bar{K}^*$	$3.12 \sim 6.20$	$23.10 \sim 44.40$
$\omega\omega$	$1.15 \sim 2.28$	$9.33 \sim 17.90$

TABLE III: The DOZI-evading contributions to the partial widths of $\eta_c(\eta_c') \to VV$ estimated by the charmed meson loops. The parameter α varies between $0.69 \sim 0.78$, which produces about 10% of the experimental data for $\eta_c \to VV$ as estimated by the parametrization scheme.

i) In case that the decays of η_c $(\eta_c') \to h$ to be dominated by the short-distance transitions, i.e. SOZI process, one expects that the relation of Eq. (3) would be respected, i.e.

$$\Gamma_{\eta_c' \to VV} \simeq \left| \frac{\eta_c'(0)}{\eta_c(0)} \right|^2 \times \Gamma_{\eta_c \to VV} \simeq 0.41 \times \Gamma_{\eta_c \to VV} \ , \tag{19}$$

where the small DOZI-evading contributions in $\eta_c \to VV$ are neglected.

ii) In case that the DOZI-evading contributions in $\eta_c \to VV$ amount to about 10% of the SOZI as shown by the fitting results of the previous section, it shows that the intermediate charmed meson loops will be sizeable in $\eta_c' \to VV$. In Table III, we list the DOZI-evading contributions to the partial widths in comparison with the SOZI-dominant predictions. These two mechanisms seem to be compatible in η_c', which may easily violate the relation of Eq. (3). Such an effect can be directly examined by experimental data for the $\eta_c' \to VV$. Such a possibility that the DOZI-evading contributions are sizeable in $\eta_c' \to VV$ exhibits similarities as in $\psi' \to VP$, where the charmed meson loops play an important role of causing the deviations from the "12% rule".

iii) It should be mentioned again that the decays $\eta_c(\eta_c') \to VV$ are strong-interaction-dominant processes, which are different from the decays $J/\psi(\psi') \to VP$ where the EM interaction will also play an important role, especially in $\psi' \to VP$ [17, 18]. We illustrate this point in Fig. 6. By these diagrams, a naive power counting indicates that

$$\begin{aligned} \frac{T_{em}}{T_{str}} &\sim \frac{\alpha_e}{\alpha_s^2} \ \text{ for } \ J/\psi(\psi') \to VP, \\ \frac{T_{em}}{T_{str}} &\sim \alpha_e \ \text{ for } \ \eta_c(\eta_c') \to VV, \end{aligned} \tag{20}$$

where "T_{em}" and "T_{str}" denote the leading EM and strong transition amplitudes, respectively. It implies that the EM contribution in $\eta_c(\eta_c') \to VV$ will be less important than that in $J/\psi(\psi') \to VP$.

An interesting expectation that distinguishes the role played by the EM transitions in $\eta_c(\eta_c') \to VV$ and $J/\psi(\psi') \to VP$ is that the long-distance corrections from the charmed meson loops should have the same relative phases in different exclusive decay channels. This means that the possible deviations from Eq. (19) caused by the charmed meson loops will either enhance or lower the overall $\eta_c' \to VV$ branching ratios. In contrast, in $J/\psi(\psi') \to VP$, interferences due to the EM transitions may have different phases in different channels, e.g. the branching ratios of $\psi' \to K^{*0}\bar{K}^0 + c.c.$ is much enhanced in comparison with $\psi' \to K^{*+}K^- + c.c.$ [1]. Therefore, observation of systematic deviations from Eq. (19) would be a strong evidence of contributions from intermediate charmed meson loops.

IV. SUMMARY

In this paper, we have investigated the processes of $\eta_c(\eta_c') \to VV$. These decay modes are supposed to be highly suppressed by the HSR but found to be rather important according to the available data. The intermediate charmed meson loop transitions, which are correlated with the DOZI-evading, are introduced to provide a mechanism for the HSR violation, and the results indicate that the effect of these loops may be significant, especially in $\eta_c' \to VV$.

Although we are still lack of experimental information to constrain the model parameters in these transitions, we show that a parametrization scheme based on the unique VVP coupling structure will allow us to parameterize out the SOZI and DOZI-evading transitions in $\eta_c \to VV$. By assuming that the DOZI-evading processes can be recognized by the intermediate charmed meson loops as a soft long-distance transition, we find that the SOZI contributions in $\eta_c \to VV$ can be well constrained by the experimental data, and an upper limit for the DOZI-evading contributions

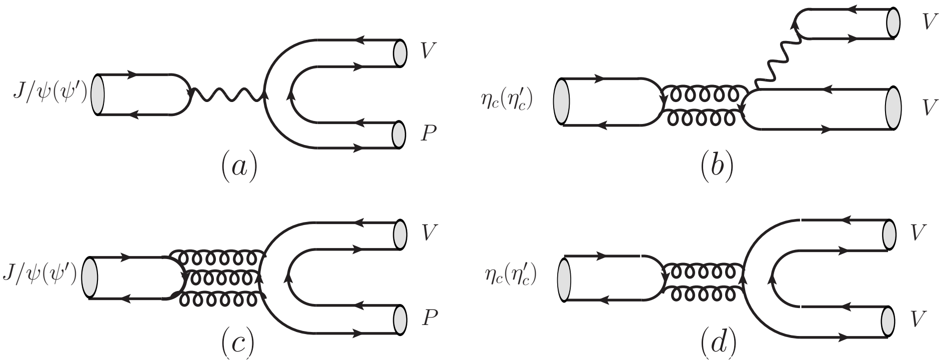

FIG. 6: Schematic diagrams for the EM and strong decays of $J/\psi(\psi') \to VP$ and $\eta_c(\eta_c') \to VV$.

can be set. This allows us to proceed and estimate the DOZI-evading contributions in $\eta_c' \to VV$. It shows that the intermediate charmed meson loops may produce measurable effects in $\eta_c' \to VV$ and cause sizeable deviations from the relation predicted by the dominance of the SOZI transitions.

This situation is similar to the exclusive processes of $J/\psi(\psi') \to VP$. There, the intermediate charmed meson loops provide a natural explanation for the so-called "$\rho\pi$ puzzle". In this sense, the study of $\eta_c(\eta_c') \to VV$ would provide additional evidence for the intermediate charmed meson loops in charmonium decays, and can be examined quantitatively by the future BESIII experiment [39].

We should also mention that our conclusions are based on the hypothesis that the flavor components of η_c and η_c' are dominated by $c\bar{c}$. Thus, the connection of their spatial wavefunctions with those of J/ψ and ψ' will make sense. If η_c or η_c' possesses some other internal structures [6, 7, 40], the relation between their branching ratios will be affected to some extent, which however, is not our focus in this work.

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